# Geometry-aware Control and Learning in Robotics

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### I. RESEARCH QUESTIONS

In many robotic applications, the encountered data have specific geometric properties. Examples range from unit quaternions to represent orientation to multidimensional arrays found in sensing and control applications. Of particular interest, symmetric positive definite (SPD) matrices are widely encountered in robotics in the form of manipulability ellipsoids [21], inertia matrices or sensory information in the form of spatial covariances [20]. The common practice to apply machine learning or control algorithms to SPD data is first to vectorize them, use them in the learning or control process, and reshaping them back into matrices, possibly requiring postprocessing such as scaling to satisfy the desired geometric properties of SPD matrices. Such a procedure typically results in a loss of precision.

In my research, I address the following question: how can we improve the control and learning in robotics by considering more tightly the underlying structure and geometry of the data? For this purpose, the aim is to extend statistical regression methods so that they handle particular geometries. I focus on SPD data as they are widely used and necessary in applications such as manipulability ellipsoids tracking and transfer, or hand movement tracking from surface electromyography (sEMG) data, processed as spatial covariances.

#### II. RESEARCH APPROACH

In order to develop geometry-aware control and learning methods, we have to consider the fact that the set of  $D \times D$ SPD matrices  $S_{++}^D$  is not a vector space since it is not closed under addition and scalar product [15], and thus the use of classical Euclidean space methods for treating and analyzing these matrices is inadequate. A compelling solution to handle this kind of data is to endow these matrices with a Riemannian metric so that these form a Riemannian manifold. Intuitively, a Riemannian manifold  $\mathcal{M}$  is a mathematical space for which each point locally resembles a Euclidean space. For each point  $x \in \mathcal{M}$ , there exists a tangent space  $\mathcal{T}_x \mathcal{M}$  equipped with a positive definite inner product. Back and forth mapping between the manifold and its tangent space allows a geometricaware processing of the data, while keeping classical tools for processing in the tangent space (an Euclidean space), allowing us to extend statistical regression methods to SPD data.

Moreover, to exploit the structure of the data and avoid loss of information due to vectorization of matrices, I explore in my research how to exploit tensor representations and multilinear algebra methods. Tensors are generalization of matrices to arrays of higher dimensions [12], where vectors and matrices may respectively be seen as 1st and 2nd-order tensors. Tensor representations permit to represent and exploit *a priori* data structure of multidimensional arrays.

In my current work, I exploit Riemannian geometry and tensor methods to track manipulability ellipsoid in a geometryaware control framework. I am also exploring the use of those tools to generalize existing probabilistic learning methods to SPD manifold, and exploit them for manipulability ellipsoids learning and for predicting hand movements from sEMG data for the control of prosthetic hands.

## III. RELATED WORK, CURRENT PROGRESS AND EVALUATION

The manipulability ellipsoid [21] serves as a geometric descriptor that indicates the ability to arbitrarily perform motion and exert a force along the different task directions in a given joint configuration. Several authors proposed to use the manipulability ellipsoids to improve trajectory generation [5, 4, 8, 19]. Note that the aforementioned approaches do not specify a desired robot manipulability for the task. Moreover, other approaches proposed to predetermine the desired robot manipulability to achieve a given task [13, 14]. However, all the aforementioned approaches overlooked an important characteristic of manipulability ellipsoids, namely, the fact that they lie on the manifold of SPD matrices. This may potentially influence the optimal robot joint configuration for the task at hand.

We propose to address the problem of tracking robot manipulability ellipsoids from a novel geometry-aware control perspective. Given a desired profile of manipulability ellipsoids, the goal of the robot is to adapt its posture to match the desired manipulability, either as its main task or as a secondary objective. Our work extends the classical inverse kinematics problem to manipulability ellipsoids, by establishing a mapping between a change of manipulability ellipsoid and the robot joint velocity, which permits to compute the desired robot joint values that lead the robot to track a desired manipulability ellipsoid [10]. To do so, we exploit tensor representation and Riemannian manifolds to obtain a geometry-aware manipulability tracking controller. This enables the robot to modify its posture so that its manipulability ellipsoid matches a desired one, either as a main control task or as a redundancy resolution problem where the manipulability tracking is viewed as a secondary objective. We showed that the proposed formulation outperforms previous gradientbased approaches and provides a faster convergence rate. Furthermore, we showed that our approach is compatible with

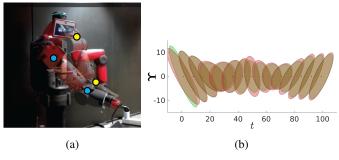


Fig. 1: (*a*) Insertion task: the poses of the robot obtained with and without manipulability tracking are respectively depicted by yellow and blue dots on the elbow and wrist bend joints. We can see that the adopted posture is different to let the manipulability coincide as accurately as possible with the desired force manipulability ellipsoid. This variation of the joint configuration consequently allows the robot to adopt a posture compatible with the control force required by the task (namely, providing sufficient force along the vertical direction). (*b*) Desired and reproduced manipulability ellipsoids (in green and red, respectively) over time, given in seconds. This plot shows the manipulability ellipsoids match resulting from the proposed manipulability transfer approach.

statistical methods providing 4th-order covariances, allowing us to exploit task variations to characterize the precision of the manipulability tracking problem, with stronger tracking along low variability directions. We evaluated the performance of the proposed geometry-aware manipulability controller in two different scenarios, namely, a pushing task and a peg-in-hole task (plugging an electric cable into an power socket), achieved by the 7-DoF arm of the Baxter robot (see Fig. 1a).

In the context of learning in robotics, Gaussian mixture regression (GMR) [6] is a widely used method to generate robot motions [3], that provides a fast and efficient way to estimate multivariate output data from multivariate input data in the form of Gaussian distributions with full covariances. It exploits the Gaussian conditioning property to estimate the distribution of output data given input data, from the joint distribution of input and output datapoints estimated by a Gaussian mixture model (GMM). The approach does not learn the regression function directly, but instead relies on the learned joint distribution. Several publications presented methods for regression from a mixture of Gaussians on Riemannian manifolds, although they only partially exploit the manifold structure in Gaussian conditioning [18, 11, 17, 23]. The probabilistic encoding using Gaussians was extended to Riemannian manifolds with data represented in vector form in [22]. Based on this work, we extended the formulations of GMM and GMR to data in the form of SPD matrices, by exploiting tensor methods to characterize the (co)variability of 2nd-order tensors by a 4th-order covariance tensor and Riemannian geometry tools to consider the underlying geometric properties of SPD data [9, 16].

We experimented this approach to transfer manipulability ellipsoids to robots, allowing them to modify their posture in function of the task, and in the context of prosthetic hands, with the goal of identifying wrist movements from spatial

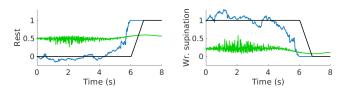


Fig. 2: Comparison between the wrist movements predicted by GMR on SPD manifold (blue line) and GMR in Euclidean space (green line) during a transition from wrist supination to rest. The reference is the black line. We can see that GMR on SPD manifold is more efficient at predicting a transition than GMR in Euclidean space, which tends to favor rest poses in most of the cases.

covariance features acquired by surface electromyography. The first problem is casted as a manipulability transfer between a teacher, who demonstrates how to perform a task with a desired time-varying manipulability profile, and a learner who reproduces the task by exploiting its own redundant kinematic structure so that its manipulability ellipsoid matches the demonstration [16]. This approach offers the possibility of transferring posture-dependent task requirements such as preferred directions for motion and force exertion in operational space, which are encapsulated in the demonstrated manipulability ellipsoids (see Fig. 1b). The second application exploits our learning approach to identify wrist movements corresponding to sEMG data in the form of spatial covariances [9]. The proposed method was tested on data from the publicly available Ninapro database [1], comprising recordings from 40 able-bodied subjects. We compared our geometryaware formulation of GMR with the standard approach in Euclidean space. It improved the detection of wrist movement for most of the subjects and proved to be efficient to detect transitions between movements (see Fig. 2).

## IV. FUTURE WORK

As future work, we plan to combine our manipulability tracking control approach with learning from demonstration techniques, using the developed manipulability transfer framework. We will explore the use of our formulation in more complex tasks involving full 6D manipulability ellipsoids, and scenarios where a humanoid robot is required to track a manipulability ellipsoid defined at either its center of mass or zero-moment point [2, 7]. The tasks of interest will be those in which velocity and force control requirements vary over the course of the task execution, which will be directly related to changes in the velocity/force manipulability ellipsoids.

In the context of prosthetic hands, we will extend the regression problem to fingers movements. We also envisage to exploit the 4th-order covariance information retrieved by GMR to regulate how precisely the hand movement is predicted and should be matched. We plan more extensive evaluation in real-time as well as tests of the approach with amputated patients. Finally, we plan to extend the proposed regression method to other Riemannian manifolds that could be exploited for data with specific underlying structure and geometry in robotics.

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