Geometry-aware Robot Manipulability Transfer

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***Abstract*—Humans naturally vary their body posture in order to quickly move or apply forces along specific directions. Such posture changes are strongly linked to the specific requirements of the task at hand, and therefore play a relevant role on task performance. Posture variation also has a significant role in robot manipulation (e.g., pushing/pulling objects, reaching tasks), where manipulability arises as a useful criterion to analyze and control the robot dexterity as a function of its joint configuration. In this context, this paper introduces a novel framework for transferring manipulability ellipsoids to robots. This framework is first built on a probabilistic learning model that allows for the geometry of the symmetric positive definite manifold to encode and retrieve appropriate manipulability ellipsoids. This geometry-aware approach is later exploited for designing a manipulability tracking controller inspired by the classical inverse kinematics problem in robotics. Experiment in simulation with planar robot arms validate the feasibility of our manipulability transfer framework.**

I. Introduction

Body posture can greatly influence human performance when carrying out manipulation tasks. Adopting an appropriate pose helps us regulate our motion and strengthen capabilities according to the task requirements. This effect is also observed in robotic manipulation where the robot joint configuration affects not only the ability to move freely in all directions in the workspace, but also the capability to generate forces along different axes. In this context, manipulability ellipsoids arise as a useful tool to analyze, control and design the robot dexterity as a function of the articulatory joints configuration.

Velocity and force manipulability ellipsoids introduced in [12] are kinetostatic performance measures of robotic platforms. They indicate the preferred directions in which force or velocity control commands may be performed at a given joint configuration. More specifically, the velocity manipulability ellipsoid describes the characteristics of feasible motion in Cartesian space corresponding to all the unit norm joint velocities. The major axis of the velocity manipulability ellipsoid $\boldsymbol{\Upsilon}^{\dot{\boldsymbol{x}}} = (\boldsymbol{J}\boldsymbol{J}^{\top})^{-1}$ indicates the direction in which the greater velocity can be generated, which is also the direction in which the robot is more sensitive to perturbations because of the duality of velocity and force (see [2] for details).

Any manipulability ellipsoid $\boldsymbol{\Upsilon}$ belongs to the set of $D \times D$ symmetric positive definite (SPD) matrices $\mathcal{S}_{++}^{D}$. Consequently, in order to properly work with manipulability ellipsoids, we must consider that the set $\mathcal{S}_{++}^{D}$ is not a vector space but forms a Riemannian manifold [8]. Intuitively, a Riemannian manifold $\mathcal{M}$ is a mathematical space for which each point locally resembles a Euclidean space. For each point $\boldsymbol{x} \in \mathcal{M}$, there exists a tangent space $\mathcal{T}_{\boldsymbol{x}}\mathcal{M}$ equipped with a positive definite inner product. In the case of the SPD manifold, the tangent space at any point $\boldsymbol{\Sigma} \in \mathcal{S}_{++}^{D}$ is identified by the space of symmetric matrices Sym^{D}. The space of SPD matrices can be represented as the interior of a convex cone embedded in its tangent space Sym^{D}. Tangent spaces allow us to manipulate data using classical Euclidean operations. To do so, we need mappings back and forth between $\mathcal{T}_{\boldsymbol{p}}\mathcal{M}$ and $\mathcal{M}$, which are known as exponential and logarithm maps ($\text{Exp}_{\boldsymbol{\Sigma}}$, $\text{Log}_{\boldsymbol{\Sigma}}$), illustrated in Fig. 1-*right*.

Moreover, as manipulability ellipsoids are represented by matrices, tensor representation is needed for different mathematical operations, such as the computation of covariance for a set of matrices. Tensors are generalization of matrices to arrays of higher dimensions [7], where vectors and matrices may respectively be seen as 1st and 2nd-order tensors. These permit to represent and exploit *a priori* data structure of multidimensional arrays. In this paper, such representation is mainly used to find the first-order differential relationship between a vector and the robot manipulability ellipsoid (3rd-order tensor) or to compute the covariance of a set of manipulability ellipsoids (4th-order tensor).

We introduce the novel idea that manipulability-based posture variation for task compatibility can be addressed from a robot learning from demonstration perspective. Specifically, we cast this problem as a *manipulability transfer* between a teacher and a learner. The former demonstrates how to perform a task with a desired time-varying manipulability profile, while the latter reproduces the task by exploiting its own redundant kinematic structure so that its manipulability ellipsoid matches the demonstration. Unlike classical learning frameworks that encode reference position, velocity and force trajectories, our approach offers the possibility of transferring posture-dependent task requirements such as preferred directions for motion and force exertion in operational space, which are encapsulated in the demonstrated manipulability ellipsoids.

This paper couples the two main challenges addressed in our previous publications, namely *(i)* encoding and retrieving manipulability ellipsoids [10], and *(ii)* tracking of robot manipulability [6]. To address the former problem, we propose a tensor-based formulation of Gaussian mixture model (GMM) and Gaussian mixture regression (GMR) that take into

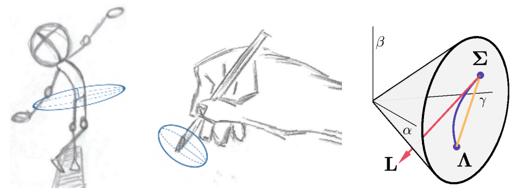

Fig. 1: *Left:* Examples of applications where the manipulability ellipsoid (in blue) is closely linked to the dexterity of the executed movement. *Right:* Representation of the SPD manifold $\mathcal{S}^2_{++}$ embedded in its tangent space Sym^2. One point on the graph corresponds to a matrix $\begin{pmatrix} \alpha & \beta \\ \beta & \gamma \end{pmatrix} \in \text{Sym}^2$. Points inside the cone, such as $\boldsymbol{\Sigma}$ and $\boldsymbol{\Lambda}$, belong to the manifold. $\boldsymbol{L}$ lies on the tangent space of $\boldsymbol{\Sigma}$ such that $\boldsymbol{L} = \text{Log}_{\boldsymbol{\Sigma}}(\boldsymbol{\Lambda})$. The shortest path between $\boldsymbol{\Sigma}$ and $\boldsymbol{\Lambda}$ is the geodesic represented as a purple curve in the graph. Note that it does not correspond to the Euclidean path, depicted by the yellow line.

account that manipulability ellipsoids lie on the manifold of SPD matrices (see Section II). The latter challenge is solved through a manipulability tracking formulation where a first-order differential relationship between the robot manipulability ellipsoid and the robot joints is established. This approach is inspired by the classical inverse kinematics problem in robotics and exploits tensor-based representations and differential geometry in order to take into account the geometric properties of the manipulability ellipsoids (see Section III). We combine and evaluate the two aforementioned solutions in a simulated tracking task where a planar robot is required to track a Cartesian position trajectory and a time-varying desired manipulability profile (see Section IV).

II. Learning Manipulability Ellipsoids

The first open problem in manipulability transfer is to appropriately encode and retrieve manipulability ellipsoids. In order to describe how we tackle this problem, we summarize the mathematical formulation of a GMM that encodes a distribution of manipulability ellipsoids over the manifold of SPD matrices. After, we describe how desired manipulability ellipsoids can be retrieved via GMR acting on the SPD manifold. The complete formulation can be found in [10].

Similarly to multivariate distribution (see [13, 11, 3]), a tensor-variate distribution maximizing the entropy in the tangent space is approximated by

$$\mathcal{N}_{\mathcal{M}}(\boldsymbol{X}|\boldsymbol{M},\boldsymbol{\mathcal{S}}) = \frac{1}{\sqrt{(2\pi)^{\tilde{D}}|\boldsymbol{\mathcal{S}}|}}\, e^{-\frac{1}{2}\text{Log}_{\boldsymbol{X}}(\boldsymbol{M})\,\boldsymbol{\mathcal{S}}^{-1}\,\text{Log}_{\boldsymbol{X}}(\boldsymbol{M})}, \tag{1}$$

where $\boldsymbol{X} \in \mathcal{T}_{\boldsymbol{M}}\mathcal{M}$, $\boldsymbol{M} \in \mathcal{M}$ is the origin in the tangent space and $\boldsymbol{\mathcal{S}} \in \mathcal{T}_{\boldsymbol{M}}\mathcal{M}$ is the covariance tensor.

Similarly to the Euclidean case, a GMM on the SPD manifold is defined by

$$p(\boldsymbol{X}) = \sum_{k=1}^{K} \pi_k \mathcal{N}_{\mathcal{M}}(\boldsymbol{X}|\boldsymbol{M}_k, \boldsymbol{\mathcal{S}}_k), \tag{2}$$

with K being the number of components of the model, and π_k representing the priors such that $\sum_k \pi_k = 1$. The parameters of the GMM on the manifold are estimated by Expectation-Maximization (EM) algorithm as described in [5, 10].

GMR computes the conditional distribution $p(\boldsymbol{X}_{\mathcal{OO}}|\boldsymbol{X}_{\mathcal{II}})$ of the joint distribution $p(\boldsymbol{X})$, where the sub-indices $\mathcal{I}$ and $\mathcal{O}$ denote the sets of dimensions that span the input and output variables. Similarly to GMR in Euclidean space [9] and in manifolds where data are represented by vectors [13], GMR on SPD manifold approximates the conditional distribution by a single Gaussian

$$p(\boldsymbol{X}_{\mathcal{OO}}|\boldsymbol{X}_{\mathcal{II}}) \sim \mathcal{N}(\hat{\boldsymbol{M}}_{\mathcal{OO}}, \hat{\boldsymbol{\mathcal{S}}}^{\mathcal{OO}}_{\mathcal{OO}}), \tag{3}$$

where the expected output mean $\hat{\boldsymbol{M}}_{\mathcal{OO}}$ is computed iteratively until convergence in its tangent space and the expected output covariance $\hat{\boldsymbol{\mathcal{S}}}^{\mathcal{OO}}_{\mathcal{OO}}$ is then computed in the tangent space of the mean (see [5, 10] for details).

III. Geometry-aware Tracking of Manipulability Ellipsoids

Several manipulation tasks in robotics may demand the robot to track a desired trajectory with a specific velocity profile, or apply forces along different task axes. These requirements are more easily achievable if the robot finds an appropriate posture that permits to apply the required velocity or force control commands. This problem can be viewed as matching a set of desired manipulability ellipsoids that are compatible with the task requirements, so that the robot performs successfully. In this section, we introduce an approach that addresses this problem. The detailed formulation can be found in [6].

Given a desired profile of manipulability ellipsoids, the goal of the robot is to vary its posture to match the desired manipulability, either as its main task or as a secondary objective. We here propose a formulation inspired by the classical inverse kinematics problem in robotics, which permits to compute the desired robot joint values that lead the robot to match a desired manipulability ellipsoid.

First, let us write the manipulability ellipsoid as a function of time as $\boldsymbol{\Upsilon}(t) = f(\boldsymbol{J}(\boldsymbol{q}(t)))$, for which we can compute the first-order time derivative by applying the chain rule as follows

$$\frac{\partial \boldsymbol{\Upsilon}(t)}{\partial t} = \frac{\partial f(\boldsymbol{J}(\boldsymbol{q}))}{\partial \boldsymbol{q}} \times_3 \frac{\partial \boldsymbol{q}(t)}{\partial t}^{\top} = \boldsymbol{\mathcal{J}}(\boldsymbol{q}) \times_3 \dot{\boldsymbol{q}}^{\top}, \tag{4}$$

where $\boldsymbol{\mathcal{J}} \in \mathbb{R}^{6\times6\times n}$ is the manipulability Jacobian and represents the linear sensitivity of the changes in the robot manipulability ellipsoid $\dot{\boldsymbol{\Upsilon}} = \frac{\partial \boldsymbol{\Upsilon}(t)}{\partial t}$ to the joint velocity $\dot{\boldsymbol{q}} = \frac{\partial \boldsymbol{q}(t)}{\partial t}$, and $\times_n$ is the n-mode product. Note that the computation of the manipulability Jacobian depends on the type of manipulability ellipsoid that is used [6].

The standard robot control approach to track a desired end-effector trajectory is to compute joint velocity commands using the inverse kinematics formulation derived from $\dot{\boldsymbol{x}} = \boldsymbol{J}(\boldsymbol{q})\dot{\boldsymbol{q}}$. We here use a similar approach to compute the joint velocities $\dot{\boldsymbol{q}}$ to track a desired manipulability profile. More specifically, by minimizing the L-2 norm of the residuals $\min_{\dot{\boldsymbol{q}}} \|\dot{\boldsymbol{\Upsilon}} - \boldsymbol{\mathcal{J}} \times_3 \dot{\boldsymbol{q}}^{\top}\|$, we can compute the required joint velocities to track a profile of desired manipulability ellipsoids as its main task with $\dot{\boldsymbol{q}} = (\mathcal{J}^{\dagger}_{(3)})^{\top}\text{vec}(\dot{\boldsymbol{\Upsilon}})$.

The formulation allows us to define a controller aimed at tracking a reference manipulability ellipsoid as main task, similarly as the classical velocity-based control that tracks a desired task-space velocity. Alternatively, for the case in which the main task of the robot is to follow reference trajectories such as Cartesian positions or force profiles, the tracking of a profile of manipulability ellipsoids is assigned a secondary role. Hence, the robot task objectives are to track the reference trajectories while exploiting its redundancy to maximize the match between the current manipulability ellipsoid and the desired one. In this situation, a manipulability-based redundancy resolution is carried out by computing a null-space velocity computed from the manipulability tracking controller defined above. Therefore, for the case of tracking a desired Cartesian position $\hat{\boldsymbol{x}}_t$ as main task, the full control law is given by

$$\dot{\boldsymbol{q}}_t = \boldsymbol{J}^{\dagger}\boldsymbol{K}_{\boldsymbol{x}}(\hat{\boldsymbol{x}}_t - \boldsymbol{x}_t) + (\boldsymbol{I} - \boldsymbol{J}^{\dagger}\boldsymbol{J})(\mathcal{J}^{\dagger}_{(3)})^{\top}\boldsymbol{K}_{\boldsymbol{M}}\text{vec}(\text{Log}_{\boldsymbol{\Upsilon}_t}(\hat{\boldsymbol{\Upsilon}}_t)). \tag{5}$$

where $\boldsymbol{K}_{\boldsymbol{M}}$ is a gain matrix. Note that the proposed formulation outperforms previous non-geometry-aware or gradient-based approaches and provides a faster convergence rate [6].

IV. Experiments

For the demonstration phase, a 3-DOF *teacher* robot follows a C-shape trajectory four times, from which we extracted both the end-effector position $\boldsymbol{x}_t$ and robot manipulability ellipsoid $\boldsymbol{\Upsilon}_t(\boldsymbol{q})$, at each time step t. The collected time-aligned data was split into two training datasets of time-driven trajectories, namely Cartesian position and manipulability. We trained a classical GMM over the time-driven Cartesian trajectories and a geometry-aware GMM over the time-driven manipulability ellipsoids, using models with five components, i.e. $K = 5$ (the number was selected by the experimenter).

During the reproduction phase, a 5-DOF *student* robot executed the time-driven task by following a desired Cartesian trajectory $\hat{\boldsymbol{x}}_t$ computed from a classical GMR as $\hat{\boldsymbol{x}}_t \sim \mathcal{P}(\boldsymbol{x}|t)$. As secondary task, the robot was also required to vary its joint configuration for matching desired manipulability ellipsoids $\hat{\boldsymbol{\Upsilon}}_t \sim \mathcal{P}(\boldsymbol{\Upsilon}|t)$, estimated by GMR over the SPD manifold. The robot implemented the geometry-aware controller defined by (5). The gain $\boldsymbol{K}_{\boldsymbol{M}}$ was defined either as a scalar value or as a diagonal matrix composed of the diagonal components of a precision tensor, namely the inverse of the covariance tensor $\hat{\boldsymbol{\mathcal{S}}}^{\mathcal{OO}}_{\mathcal{OO}}$ retrieved by GMR. Our goal here was to exploit the learned variability information of the task to demand the robot a high precision tracking where low variability is observed, and vice-versa.

Figure 2a shows the four demonstrations carried out by the 3-DOF robot, where both the Cartesian trajectory and manipulability ellipsoids are displayed. Note that the recorded manipulability ellipsoids slightly change across demonstrations as a side effect of the variation observed in both the initial end-effector position and the generated trajectory. Figure 2b displays the demonstrated ellipsoids (in gray) along with the

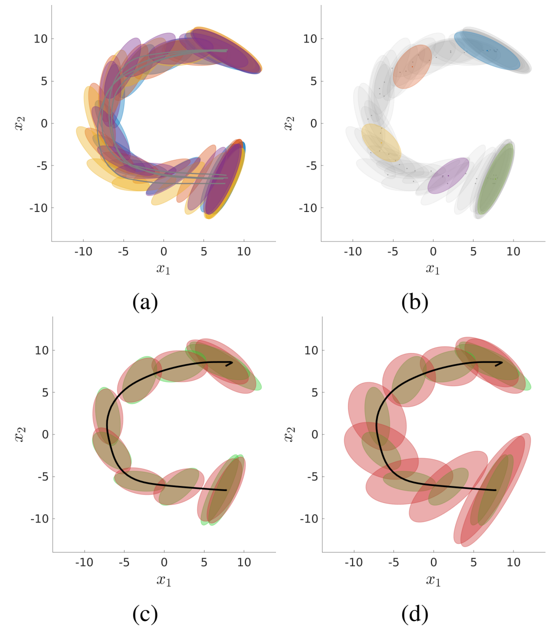

Fig. 2: (*a*) Four demonstrations of a 3-DOF planar robot tracking a C-shape trajectory. The path followed by the robot end-effector (light gray solid lines) and the manipulability ellipsoids at different time steps are represented. (*b*) Demonstrated manipulability ellipsoids(in gray) and centers $\boldsymbol{M}_k$ of the 5-states GMM in the SPD manifold. (*c*), (*d*) Reproductions of a C-shape tracking task (Cartesian trajectory in black solid line, desired and reproduced manipulability ellipsoids in green and red, respectively).

center $\boldsymbol{M}_k$ of the five components of the GMM encoding $\boldsymbol{X}^{\boldsymbol{\Upsilon}}$. These are centered at the Cartesian position recovered by GMR for the time steps represented in the GMM encoding $\boldsymbol{X}^{\boldsymbol{\Upsilon}}$. A successful reproduction of the tracking task using our manipulability-based redundancy resolution controller with a scalar gain and a variability-based matrix gain are shown in Figures 2c and 2d, respectively. Note that the variability-based matrix gain changes the required precision tracking, where higher precision is required at both the beginning and the end of the task. These results validate that the proposed approach allows the robot to learn and reproduce reference trajectories, while fulfilling additional task requirements encapsulated in a profile of desired manipulability ellipsoids.

V. Conclusions and Future Work

This paper presented a novel approach to learn and track robot manipulability ellipsoids. Our work exploits tensor representation and Riemannian manifolds to obtain a geometry-aware learning framework and tracking controller for robot manipulability. The reported results show the effectiveness of the proposed approach for transferring manipulability ellipsoids between robots that differ in their kinematic structure. As future work, we will explore the use of our formulation in more complex tasks involving full 6D manipulability ellipsoids, and scenarios where a humanoid robot is required to track a manipulability ellipsoid defined at either its center of mass or zero-moment point [1, 4].

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